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An improved model-based method to test circuit faults

Xiaochun Cheng[a, b, *], Dantong Ouyang[b, c], Jiang Yunfei[b, d], Chengqi Zhang[b, e]

[a] *Software Institute, Northeast Normal University, Changchun 130024, China*
[b] *National Laboratory of Symbolic Computation and Knowledge Engineering, Changchun 130012, China*
[c] *Department of Computer Science and Technology, Jilin University, Changchun 130012, China*
[d] *Software Institute, Zhongshan University, Guangzhou 510275, China*
[e] *Faculty of Information Technology, University of Technology, Sydney, Broadway, NSW 2007, Australia*

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Abstract

This paper presents an improved model-based reasoning method to test circuit faults. The testing procedure is applicable even when the target system contains multiple faulty modes. Using our method, the observation could be planned appropriately to guarantee correct solutions to be in the restricted candidate space. The existent consistency-checking method and abductive reasoning method are special cases of our method. The relationship between the testing procedure and the corresponding prime implication is analyzed for algorithmic implementation.

Keywords: Testing; Search; Deduction; Abduction; Prime implication

1. Introduction

When there are discrepancies between the predicted behaviors and the observed behaviors of target systems, we need to locate system faults. For approaches to build diagnostic expert systems [2], the acquisition of domain rules from human experts is a bottleneck.

* Corresponding author. Tel.: +44 7762227784.
E-mail address: x.cheng@ieee.org (X. Cheng).

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A model-based method employs the system model including the knowledge of structures and behaviors of the devices to be diagnosed. Model-based diagnosis is also called as "diagnosis from first principles" [20], or "diagnosis based on structure and behavior" [5]. The device model can be available as long as the device is designed and manufactured. The explicit descriptions in electric forms of the device models may be available if the devices are produced by CAD/CAM systems. Therefore, the model-based method can overcome the above shortcoming of the expert system approach [2]. With the wide application of many representative circuit fault diagnosis systems and a number of medical diagnosis systems [1], model-based diagnosis attracts more and more researchers and system developers [1–20].

Consistency checking [5,9,20,12] and abductive reasoning [18,3] are two representative methods for model-based diagnosis. Consistency-based diagnosis demands that the explanation should be consistent with the system model and the observation. The consistency constraint is weak. As a result, the solution space may contain irrelevant candidates. Abductive diagnosis demands that all the observations should be logically deduced from the system model. Therefore, the abductive method is applicable only when the system model is complete, i.e., explanations of each observation can be generated using the system model. If the system model is incomplete, we can use a consistency-check method.

Console and Torasso [4] reformulated the diagnosis problem as an abductive reasoning problem with consistency constraints such that most existent logical definitions of diagnosis could be represented and compared within the same framework. de Kleer et al. [11] developed kernel consistency-based diagnosis. McIlraith [15] explored kernel abductive diagnosis. We proposed kernel model-based diagnosis in [17].

All these known methods consider normal behaviors of devices [5,9,20,12] (such as using predicate *ok*) or abnormal behaviors of devices [18,3] (such as using predicate *ab*). They are applicable mainly for the purpose of locating faulty components. Choosing abnormal diagnosis or normal diagnosis depends on the system description and the diagnosis task [1]. Abnormal diagnosis applies to systems with complete descriptions of normal behaviors, such as most cases of digital circuit diagnoses. Normal diagnosis applies to systems with complete descriptions of abnormal behaviors, such as most cases of medical diagnoses.

When we know refined behavior modes of the components, such as different types of faults, we are expected to point out the faulty components, and to identify the specific faulty behaviors of the faulty components. All known existent methods are not developed for this purpose.

In this paper, we improve the model-based method to overcome the above limitations of existent methods. In Section 2, we introduce system modeling details. In Section 3, we introduce model-based testing and present a generalized definition to characterize multiple faulty modes. In Section 4, we develop kernel model-based testing based on analyzing the relationships between different testing procedures and corresponding implications. In Section 5, we summarize our results, compare with known methods, and discuss future work.

2. System modeling

Given refined description on behavior modes of components, the basic task of model-based diagnosis is to determine the components' faulty modes based on the observation and

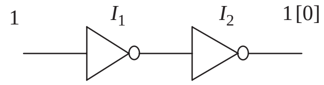

Fig. 1. A two-inverter circuit.

the behavioral and structural model of the system. The explanation should be consistent with the observation and the system model, and part of the observation should be logical results of the system model.

For a system S, $COMPS = \{C_1, \ldots, C_n\}$, where C_i $(i = 1, \ldots, n)$ is a component in S, and is characterized by a set of behavioral modes. The union of the sets of the behavioral modes for components in S is represented by a set $\boldsymbol{R}$ of abductive symbols [8]. Let $\boldsymbol{a}$ be an abductive symbol. Abductive atom $\boldsymbol{a}(C)$ denotes component C in the mode $\boldsymbol{a}$. Given the behavioral modes of the components in S, we can establish the behavioral and structural model [13] SD of S.

Definition 1. A system is formalized by a pair (SD, $COMPS$), where SD is the system description set of first-order sentences and $COMPS$ is the finite set of system component constants.

Example 1. Consider the two-inverter circuit shown in Fig. 1. $COMPS = \{I_1, I_2\}$.

The set of behavioral modes of these two inverters is $\{ok, s1, s0, short\}$. $s1$ is for "stuck at 1". $s0$ is for "stuck at 0". The following system description should be included in SD:

$$INVERTER(I_1),$$

$$INVERTER(I_2),$$

$$OUT(I_1) = IN(I_2),$$

$$IN(x) = 0 \equiv \sim(IN(x) = 1),$$

$$OUT(x) = 0 \equiv \sim(OUT(x) = 1),$$

$$INVERTER(x) \wedge ok(x) \rightarrow (IN(x) = 0 \equiv OUT(x) = 1),$$

$$INVERTER(x) \wedge s1(x) \rightarrow (OUT(x) = 1),$$

$$INVERTER(x) \wedge s0(x) \rightarrow (OUT(x) = 0),$$

$$INVERTER(x) \wedge short(x) \rightarrow (OUT(x) = IN(x)).$$

The following shows that ok, $s1$, $s0$ and $short$ are all behavioral modes of each inverter.

$$INVERTER(x) \rightarrow ok(x) \vee s1(x) \vee s0(x) \vee short(x).$$

The following shows that each behavioral mode of the inverters is excluded with other modes. For example, a diagnosis should not include both $s1(I_1)$ and $s0(I_1)$ at the same time.

$$\sim INVERTER(x) \vee \sim ok(x) \vee \sim s1(x),$$

$$\sim INVERTER(x) \vee \sim ok(x) \vee \sim s0(x),$$

$$\sim INVERTER(x) \vee \sim ok(x) \vee \sim short(x),$$

$$\sim INVERTER(x) \vee \sim s1(x) \vee \sim s0(x),$$

$$\sim INVERTER(x) \vee \sim s1(x) \vee \sim short(x),$$

$$\sim INVERTER(x) \vee \sim s0(x) \vee \sim short(x).$$

In this example, abductive symbol set $\boldsymbol{R} = \{ok, s1, s0, short\}$.

In Fig. 1, the observed value 0 is put in a square bracket. But the predicted output value is 1. The discrepancy between the observed value and the predicted value suggests faults in the system.

Definition 2. Abductive literals are $\boldsymbol{a}(C)$ and $\sim\boldsymbol{a}(C)$, where $\boldsymbol{a}(C)$ is an abductive atom. Abductive clause is a disjunction of abductive literals. Positive (negative) abductive clause is a clause with each literal in it positive (negative).

3. Model-based testing

Definition 3 (*Console and Torasso [4]*). A model-based diagnostic problem *DP* is ((*SD*, *COMPS*), *CXT*, *OBS*), where *CXT* is the context set of ground atoms and *OBS* is the observation set of ground atoms.

The difference between *CXT* and *OBS* was originally pointed out by Reggia et al. [19]. *CXT* is used to predict the behavior of the system to be diagnosed, but need not be explained by the diagnosis. *OBS* plays a different role. *OBS* must be considered for the diagnosis.

For Example 1, $CXT = \{IN(I_1) = 1\}$, $OBS = \{OUT(I_2) = 0\}$. If both I_1 and I_2 are normal, then the predicted output of I_2 is 1. But, the observed value of I_2 is 0. There exist faults in the system. The diagnosis should explain the observed value of I_2.

Definition 4. Given a system (*SD*, *COMPS*) and its abductive symbol set *R*, the conjunction *D* of ground abductive atoms is called an assignment to *COMPS*, if for each $b \in COMPS$, *D* contains just one term in the form of $\boldsymbol{a}(b)$, where $\boldsymbol{a} \in R$.

D describes a state of all components in the system. For Example 1, $ok(I_1) \wedge s0(I_2)$ is an assignment to *COMPS*, but $s0(I_1) \wedge short(I_1) \wedge ok(I_2)$ is not an assignment to *COMPS*.

Similarly, partial assignment to *COMPS* can be defined, which is an assignment to a subset of *COMPS*, and describes a state of partial components in the system.

Definition 5. Given a diagnostic problem *DP*: ((*SD*, *COMPS*), *CXT*, *OBS*), suppose $O' \subseteq OBS$, the assignment *D* to *COMPS* is called a model-based diagnosis for *DP* iff

(1) $SD \cup CXT \cup \{D\} \cup OBS$ is consistent, and

(2) if $O' \neq \phi$, then $SD \cup CXT \cup \{D\} \vdash O'$.

Assuming O' is the abnormal outputs that we are interested in, model-based diagnosis would explain the causes of these abnormal observations in O' and the explanation is consistent with $SD \cup CXT \cup OBS$. Here, diagnosis needs abductive reasoning with consistency constraints.

The existent methods [11,17] define model-based diagnosis using $D(\Delta, COMPS - \Delta)$, and consider only normal and abnormal modes, where $D(\Delta, COMPS - \Delta)$ denotes

$$\left[\bigwedge_{c\in\Delta} ab(c)\right] \wedge \left[\bigwedge_{c\in COMPS-\Delta} ok(c)\right].$$

Using our Definition 5, we can characterize multiple faulty modes of the components. When the abductive symbol set is the special case $\{ok, ab\}$, D is model-based diagnosis $D(\Delta, COMPS - \Delta)$ in Ref. [17]. When the abductive symbol set is $\{ok, ab\}$ and O' is special case ϕ, D is consistency-based diagnosis $D(\Delta, COMPS - \Delta)$ in Ref. [11]. When the abductive symbol set is $\{ok, ab\}$ and O' is OBS, D is equal to abductive diagnosis $D(\Delta, COMPS - \Delta)$ in Ref. [11]. Thus we can say that our definition is more general than known definitions.

4. Kernel model-based testing

Definition 6 (*de Kleer et al. [11]*). A conjunction G of literals covers a conjunction H of literals iff each literal of G occurs in H.

For example, $s1(I_2)$ covers $ok(I_1) \wedge s1(I_2)$.

Definition 7. Given a diagnostic problem *DP*: ((*SD*, *COMPS*), *CXT*, *OBS*), suppose $O' \subseteq OBS$. A partial model-based diagnosis for *DP* is a partial assignment D for *COMPS*, such that for each partial assignment D' covered by D, we have

(1) $SD \cup CXT \cup \{D'\} \cup OBS$ is consistent, and

(2) if $O' \neq \phi$, then $SD \cup CXT \cup \{D'\} \vdash O'$.

Partial abductive diagnosis and partial consistency-based diagnosis are two extreme cases of the above definition. Partial abductive diagnosis is the case when O' is OBS, i.e., all observations need to be explained logically. Partial consistency-based diagnosis is the case when O' is ϕ, i.e., only the consistency property is required, and logical causal relationship is not enforced.

Lemma 1. *Each partial model-based diagnosis for diagnostic problem DP is a partial consistency-based diagnosis for DP.*

The converse of Lemma 1 does not hold.

Definition 8. A kernel model-based diagnosis is a partial model-based diagnosis with the property that the only partial model-based diagnosis covering it is itself.

Kernel model-based diagnosis is minimal under covering order. Kernel model-based diagnosis can be used to compactly characterize the solution space. When O' is chosen as ϕ and *OBS*, respectively, we have definitions of kernel consistency-based diagnosis and kernel abductive diagnosis.

We will analyze the relationship between kernel model-based diagnosis and prime implication [11], in order to compute kernel model-based diagnosis.

Theorem 1. *Suppose diagnostic problem DP is* ((*SD*, *COMPS*), *CXT*, *OBS*). *Let Π be the conjunction of all prime implicates of $SD \cup CXT \cup OBS$, with each implicate in the form of a positive abductive clause. D is a kernel consistency-based diagnosis iff D is a prime implicant of Π.*

Proof (*Necessity*). Suppose D is a kernel consistency-based diagnosis of *DP*. We will prove $D \vdash \Pi$. If $D \vdash \Pi$ does not hold true, then $D \wedge \sim\Pi$ is satisfiable. By the assumption, Π is in form of $\Pi_1 \wedge \cdots \wedge \Pi_k$, where Π_i $(i = 1, \ldots, k)$ is a positive abductive clause, which is a prime implicate of $SD \cup CXT \cup OBS$. Now $\sim\Pi$ equals $\sim\Pi_1 \vee \cdots \vee \sim\Pi_k$. Since $D \wedge \sim\Pi$ is satisfiable, there must exist some clause Π_j, such that $D \wedge \sim\Pi_j$ is satisfiable. Suppose $\Pi_j = l_1 \vee \cdots \vee l_m$, then $\sim\Pi_j = \sim l_1 \wedge \cdots \wedge \sim l_m$, where l_i $(i = 1, \ldots, m)$ is an abductive atom. Suppose l_1 is in the form of $a_{11}(C)$, where a_{11} is an abductive symbol. Suppose the set of behavioral modes corresponding to C is $\{\boldsymbol{\alpha}_{11}, \boldsymbol{\alpha}_{12}, \ldots, \alpha_{1p_1}\}$. Therefore, $\sim l_1$ equals $\boldsymbol{\alpha}_{12}(C) \vee \cdots \vee \alpha_{1p_1}(C)$, which is represented as $l_{12} \vee \cdots \vee l_{1p_1}$. Consider other literals in $\sim\Pi_j$ similarly; we can see $\sim\Pi_j$ is equivalent to $(l_{12} \vee \cdots \vee l_{1p_1}) \wedge \cdots \wedge (l_{m2} \vee \cdots \vee l_{mp_m})$. Convert it into DNF and delete any conjunction containing a complementary pair of literals; thus $\sim\Pi_j$ is converted into $A_1 \vee \cdots \vee A_q$, where A_i $(i = 1, \ldots, q)$ is a satisfiable conjunction of abductive atoms. Therefore, $\sim\Pi_j$ is a partial assignment to *COMPS*. Since $D \wedge \sim\Pi_j$ is satisfiable, there must exist some conjunction A_r in $\sim\Pi_j$, such that $D \wedge A_r$ is satisfiable. There should exist some assignment D' to *COMPS*, which is covered by D, such that $D' \wedge A_r$ is satisfiable. Since A_r is a partial assignment to *COMPS*, it must be part of D'. Because D is a kernel consistency-based diagnosis, D is also a partial consistency-based diagnosis. Since D' is covered by D, $SD \cup CXT \cup OBS \cup \{D'\}$ is consistent. Therefore, $SD \cup CXT \cup OBS \cup \{A_r\}$ is consistent.

On the other hand, by the assumption, $SD \cup CXT \cup OBS \vdash \Pi$, and therefore $SD \cup CXT \cup OBS \vdash \Pi_j$. $\Pi_j = \sim(\sim\Pi_j) = \sim(A_1 \vee \cdots \vee A_r \vee \cdots \vee A_q) = (\sim A_1 \wedge \cdots \wedge \sim A_r \wedge \cdots \wedge \sim A_q)$, and therefore $SD \cup CXT \cup OBS \vdash \sim A_r$, which contradicts the fact that $SD \cup CXT \cup OBS \cup \{A_r\}$ is consistent. Thus we can say $D \vdash \Pi$, i.e. D is an implicant of Π.

In the following, we will prove that D is a prime implicant of Π. Suppose otherwise, there exists an implicant D_1 of D, such that $D_1 \vdash \Pi$, and D_1 is a sub-conjunction of D. We wish to prove $SD \cup CXT \cup OBS \cup \{D_1\}$ is consistent. If $SD \cup CXT \cup OBS \cup \{D_1\}$ is not consistent, then $SD \cup CXT \cup OBS \vdash \sim D_1$. Suppose D_1 is in the form of $l'_1 \wedge \cdots \wedge l'_n$, where l'_i $(i = 1, \ldots, n)$ is an abductive atom. Thus $\sim D_1$ is in the form of $\sim l'_1 \vee \cdots \vee \sim l'_n$. Considering negative abductive literals in $\sim D_1$ using the above methods applied to $\sim\Pi_j$, $\sim D_1$ can be converted into a disjunction B of positive abductive literals, which is in the form of $(l'_{12} \vee \cdots \vee l'_{1s1}) \vee \cdots \vee (l'_{n2} \vee \cdots \vee l'_{ns_n})$. By $SD \cup CXT \cup OBS \vdash \sim D_1$, B is an implicate of $SD \cup CXT \cup OBS$. B is in the form of a positive abductive clause. By the assumption, Π is a conjunction of all prime implicates of $SD \cup CXT \cup OBS$, with each implicate in the form of a positive abductive clause. Therefore, there must exist some clause Π_i in Π such that Π_i is a sub-disjunction of B. Since $\Pi \vdash \Pi_i$, and therefore $\Pi \vdash B$, as a result,

$\Pi \vdash \sim D_1$, which contradicts that $D_1 \vdash \Pi$. Thus we can say that $SD \cup CXT \cup OBS \cup \{D_1\}$ is consistent. Therefore, D_1 is a partial consistency-based diagnosis. Because D_1 covers kernel consistency-based diagnosis D, by definition of kernel diagnosis, D_1 is D itself. Now we can say that D is a prime implicant of Π.

(*Sufficiency*). If D is a prime implicant of Π, then by the above proof, D must be a partial consistency-based diagnosis for *DP*. Now we will prove that D is the kernel consistency-based diagnosis. Suppose there exists a partial consistency-based diagnosis D_2, which covers D. By the above proof, D_2 must be an implicant of Π. While D_2 covers the prime implicant D of Π, D_2 is D itself. Now we can say that D is a kernel consistency-based diagnosis for *DP*. □

Example 2. We use a binary full adder as the example circuit to illustrate the diagnosis process (Fig. 2).

$$
\begin{aligned}
&\text{COMPS} = \{\text{A1, A2, X1, X2, O1}\}.\\
&\text{SD} = \{\text{andg(A1)},\\
&\qquad \text{andg(A2)},\\
&\qquad \text{xorg(X1)},\\
&\qquad \text{xorg(X2)},\\
&\qquad \text{org(O1)},\\
&\qquad \text{andg(x)} \wedge \text{ok(x)} \supset \text{out(x)} = \text{and (in1(x), in2(x))},\\
&\qquad \text{xorg(x)} \wedge \text{ok(x)} \supset \text{out(x)} = \text{xor (in1(x), in2(x))},\\
&\qquad \text{org(x)} \wedge \text{ok(x)} \supset \text{out(x)} = \text{or (in1(x), in2(x))},\\
&\qquad \text{out(x1)} = \text{in2(A2)},\\
&\qquad \text{out(x1)} = \text{in1(X2)},\\
&\qquad \text{out(A2)} = \text{in1(O1)},\\
&\qquad \text{in1(A2)} = \text{in2(X2)},\\
&\qquad \text{in1(X1)} = \text{in1(A1)},\\
&\qquad \text{in2(X1)} = \text{in2(A1)},\\
&\qquad \text{out(A1)} = \text{in2(O1)},\\
&\qquad \text{in1(x)} = 0 \vee \text{in1(x)} = 1,\\
&\qquad \text{in2(x)} = 0 \vee \text{in2(x)} = 1,\\
&\qquad \text{out(x)} = 0 \vee \text{out(x)} = 1\\
&\quad \}
\end{aligned}
$$

For this example, the abductive symbol set $\boldsymbol{R} = \{ok, ab\}$.

For this example, in1(X1) = 1, in2(X1) = 0, in1(A2) = 1. From in1(X1) = in1(A1), we have in1(A1) = 1. From in2(X1) = in2(A1), we have in2(A1) = 0. The predicted output O is{out(X2) = 0, out(O_1) = 1}. We observed that {out(X2) = 1, out(O_1) = 0}. The discrepancy between the predicted behavior and the observed behavior suggests system faults.

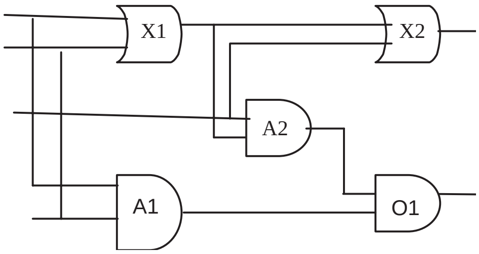

Fig. 2. A binary full adder.

There are two steps to diagnose using Theorem 1. The first step is to compute the conjunction Π of all prime implicates of $SD \cup CXT \cup OBS$. This step is domain related. The second step is to compute the prime implicants of Π. This step is domain independent.

For this example, the conjunction Π of all prime implicates of $SD \cup CXT \cup OBS$ is

$$(ab\ (\mathrm{X1}) \vee ab\ (\mathrm{X2})) \wedge (ab\ (\mathrm{X1}) \vee ab\ (\mathrm{A2}) \vee ab\ (\mathrm{O1})).$$

From the prime implicants of Π, we have following three-kernel consistency-based diagnoses:

$$ab\ (\mathrm{X1})$$

$$ab\ (\mathrm{X2}) \wedge ab\ (\mathrm{A2})$$

$$ab\ (\mathrm{X2}) \wedge ab\ (\mathrm{O1})$$

Based on the proof of Theorem 1, we can prove the following theorem similarly.

Theorem 2. *Suppose diagnostic problem DP is* $((SD, COMPS), CXT, OBS)$. *Let Π be the conjunction of all prime implicates of $SD \cup CXT \cup OBS$, with each implicate in the form of a positive abductive clause. D is a partial consistency-based diagnosis for DP iff D is an implicant of Π.*

Suppose $OBS = \{O_1, \ldots, O_n\}$, $O' \subseteq OBS$, $O' = \{O_{11}, \ldots, O_{1r}\}$.$\sim OBS$ is used to denote $\{\sim O_1 \vee \cdots \vee \sim O_n\}$, and $\sim O'$ is used to denote $\{\sim O_{11} \vee \cdots \vee \sim O_{1r}\}$.

Theorem 3. *Suppose diagnostic problem DP is* $((SD, COMPS), CXT, OBS)$. $O' \subseteq OBS$. *Let Π be the conjunction of all prime implicates of $SD \cup CXT \cup OBS$, with each implicate in the form of a positive abductive clause. Let Ω be the conjunction of all prime implicates of $SD \cup CXT \cup \sim O'$, with each implicate in the form of a negative abductive clause. If $O' = \phi$, let $\Omega = false$. D is a partial model-based diagnosis of DP iff D is an implicant of $\Pi \wedge \sim\Omega$.*

Proof (*Necessity*). Suppose D is a partial model-based diagnosis of DP; now we want to prove that $D \vdash \Pi \wedge \sim\Omega$. By Lemma 1, D is a partial consistency-based diagnosis of DP. By Theorem 2, we have $D \vdash \Pi$. Now we will prove $D \vdash \sim\Omega$. Suppose $D \vdash \sim\Omega$ does not hold, i.e., $D \nvdash \sim\Omega$. Assume that Ω is in the form of $B_1 \wedge \cdots \wedge B_s$, where B_i $(i = 1, \ldots, s)$ is a prime implicate of $SD \cup CXT \cup \sim O'$ and in the form of a negative abductive clause. Thus, $\sim\Omega$ is in the form of $\sim B_1 \vee \cdots \vee \sim B_s$. $D \nvdash \sim\Omega$, and therefore, for each $\sim B_j$ in $\sim\Omega$, $D \nvdash \sim B_j$. On the other hand, by the definition of partial model-based diagnosis, $SD \cup CXT \cup \{D\} \vdash O'$, i.e., $SD \cup CXT \cup \sim O' \vdash \sim D$. Hence $\sim D$ is an implicate of $SD \cup CXT \cup \sim O'$ in the form of a negative abductive clause. By the definition of prime implicate, there must exist some B_k in Ω such that B_k is a sub-disjunction of $\sim D$, i.e., $\sim B_k$ is a sub-conjunction of D, as a result, $D \vdash \sim B_k$, which contradicts the above result that for each $\sim B_j$ in $\sim\Omega$, $D \nvdash \sim B_j$. Thus, $D \vdash \sim\Omega$. Now we can see that $D \vdash \Pi \wedge \sim\Omega$.

(*Sufficiency*). Suppose D is an implicant of $\Pi \wedge \sim\Omega$, i.e., $D \vdash \Pi \wedge \sim\Omega$. Therefore, we have $D \vdash \Pi$. By Theorem 2, D is a partial consistency-based diagnosis for DP. Thus, $SD \cup CXT \cup \{D\} \cup OBS$ is consistent. Since $SD \cup CXT \cup \sim O' \vdash \Omega$, $SD \cup CXT \cup \sim\Omega \vdash O'$. We can prove $D \vdash \sim\Omega$, and therefore $SD \cup CXT \cup \{D\} \vdash O'$. Now we can say that D is a partial model-based diagnosis for DP. □

Theorem 4. *Suppose diagnostic problem DP is* ((*SD*, *COMPS*), *CXT*, *OBS*). $O' \subseteq OBS$. *Let Π be the conjunction of all prime implicates of $SD \cup CXT \cup OBS$, with each implicate in the form of a positive abductive clause. Let Ω be the conjunction of all prime implicates of $SD \cup CXT \cup \sim O'$, with each implicate in the form of a negative abductive clause. If $O' = \phi$, then let $\Omega = false$. D is a kernel model-based diagnosis of DP iff D is a prime implicant of $\Pi \wedge \sim\Omega$.*

Proof (*Necessity*). If D is a kernel model-based diagnosis, then it must be a partial model-based diagnosis. By Theorem 3, D is an implicant of $\Pi \wedge \sim\Omega$. Now we will prove D is prime. Otherwise, suppose there exists an implicant D' of $\Pi \wedge \sim\Omega$, covering D and different from D. By Theorem 3, D' is a partial model-based diagnosis, which contradicts the result that D is a kernel model-based diagnosis. Therefore, D is a prime implicant of $\Pi \wedge \sim\Omega$.

(*Sufficiency*). Suppose D is a prime implicant of $\Pi \wedge \sim\Omega$. By Theorem 3, D must be a partial model-based diagnosis of DP. Now we will prove that D is a kernel. Otherwise, suppose there exists a partial model-based diagnosis D' covering D and different from D. By Theorem 3, we can see that D' is an implicant of $\Pi \wedge \sim\Omega$, which contradicts the result that D is a prime implicant of $\Pi \wedge \sim\Omega$. Therefore, D is a kernel model-based diagnosis for DP. □

When we use abductive symbols $\{ok, ab\}$, we can have partial model-based diagnosis in Ref. [17] as a special case of Theorem 3 and kernel model-based diagnosis in Ref. [17] as a special case of Theorem 4. When we choose O' as ϕ and OBS, we have specialized characterizations for kernel consistency-based diagnosis and kernel abductive diagnosis, respectively.

Based on the above analyzed relationships between testing procedures and corresponding prime implications, we could incorporate ATMS [10] algorithms for testing.

Example 3. Consider Example 1 with corresponding system description *SD* given in Section 2. Suppose $CXT = \{IN(I_1) = 1\}$, $OBS = \{OUT(I_2) = 0\}$. Choose $O' = OBS$, then $\sim O' = \{OUT(I_2) = 1\}$.

All previous known diagnosis methods are designed to judge only normal and abnormal modes for components. In Example 1, we consider behavior modes ok, $s1$, $s0$ and $short$; therefore, we cannot apply previous known methods for this example. But, we can use the method presented in this paper to compute diagnoses for this example.

Firstly, we compute Π, Ω and $\sim\Omega$:

$$
\begin{aligned}
&\Pi:\\
&\quad (s0(I_1) \vee s1(I_1) \vee short(I_1) \vee s0(I_2) \vee s1(I_2) \vee short(I_2))\\
&\wedge (ok(I_2) \vee s0(I_2) \vee short(I_2))\\
&\wedge (ok(I_2) \vee s0(I_2) \vee s1(I_2) \vee ok(I_1) \vee s0(I_1) \vee short(I_1))\\
&\wedge (ok(I_2) \vee s0(I_2) \vee s1(I_2) \vee ok(I_1) \vee s0(I_1) \vee s1(I_1))\\
&\Omega:\\
&\quad (\sim s0(I_2))\\
&\wedge (\sim ok(I_1) \vee \sim short(I_2))\\
&\wedge (\sim s0(I_1) \vee \sim short(I_2))\\
&\wedge (\sim s1(I_1) \vee \sim ok(I_2))\\
&\wedge (\sim short(I_1) \vee \sim ok(I_2))\\
&\sim\Omega:\\
&\quad s0(I_2)\\
&\vee (ok(I_1) \wedge short(I_2))\\
&\vee (s0(I_1) \wedge short(I_2))\\
&\vee (s1(I_1) \wedge ok(I_2))\\
&\vee (short(I_1) \wedge ok(I_2))
\end{aligned}
$$

Secondly, we compute the prime implicants of $\Pi \wedge \sim\Omega$:

$$s0(I_2),$$

$$ok(I_1) \wedge short(I_2),$$

$$s0(I_1) \wedge short(I_2),$$

$$s1(I_1) \wedge ok(I_2),$$

$$short(I_1) \wedge ok(I_2).$$

Finally, apply Theorem 4, and we can see that the above prime implicants of $\Pi \wedge \sim\Omega$ are the kernel model-based diagnoses for Example 1, which described the refined fault behavior modes of the components.

5. Summary

All observations need to be logically explained using the system model for abductive testing. Abductive diagnosis is applicable only when the system model is complete, i.e., when we could choose $O' = OBS$ using Theorem 4. When $O' = \phi$, only consistency checking is required in Definition 7. Consistency-based diagnosis without requirements on causal relations between observations and the system model would have larger candidate solution space than that of abductive diagnosis. When the system model is incomplete, we may not be able to generate all causal relations between the observations and the system model, and as a result, abductive diagnosis may not be able to find the solution. Normally, we analyze dependency relations of system substructures in order to know which partial observation $O' \subseteq OBS$ could be explained by the system model; then we could use Theorem 4 to locate faults. In this way, we can ensure having the right solutions, and also could remove irrelevant candidates using abductive reasoning based on $O' \subseteq OBS$.

Existent kernel consistency-based method [11], kernel abductive method [15] and kernel model-based method [17] are special cases of our method. Thus, our method is more general than known existent methods. Our method can handle multiple faulty modes. Using our method, after analyzing the completeness of the system model w.r.t. observable atoms, we could choose the observation subset appropriately, in order to have restricted search space and also to ensure having the right solutions.

In future, we will incorporate our method with replacement action [14] for hierarchical system testing [16].

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